Visualizing relevant uncertain information for decision making: Fuzzy surface case study

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Abstract. Data with uncertainty are quite common and there are many models to produce approximations of the actual result. If uncertain data are used as inputs for the analyses it is only logical that the result can not be precise. The assement of the uncertainty of the result based on the uncertainties of inputs is done by process called the uncertainty propagation. For decision making it is crucial to provide user with complete information in a suitable form for the best information perception. However cartographic visualizations often fail to do so, because they often presents the data in such form, that may lead the user to conclusion that the data are actually two dimensional, with one dimension being the attribute and the second uncertainty. Proposed approach is focused on the visualization of fuzzy surfaces with regard to the uncertainty propagation. The proposed approach significantly improves the readability of the information about fuzzy surface.

Keywords: uncertainty, fuzzy surface, visualization

1 Introduction

On the basis of developments in cartography and GIS in recent decades, geovisualizations are no longer perceived as a simple tool to communicate information that is already known. Geovisualizations began to be considered as a tool for understanding the unknown relationships of any information with spatial character. In geosciences data with uncertainty are quite common. Also, a good number of models that are used for predictions are known to produce approximations of the actual result. Both these factors and some others affect the precision and certainty of outcome from any spatial analyses. As noted in [13] practically all the spatial data contain some amount of uncertainty. If uncertain data are used as inputs for the analyses it is only logical that the result can not be precise. An assement of the uncertainty of the result based on the uncertainties of inputs is done by process called the uncertainty propagation [8]. The uncertainty propagation is also called numerics of the uncertainty theory and it is specific for each of uncertainty theory [11]. There are several theories for modelling uncertainty, these include but are not limited to probability theory and statistics, fuzzy set theory, possibility theory, interval arithmetic and evidence theory. Each of these methods conceptualizes the uncertainty in a different way, utilizes different methods to propagate uncertainty through the model and each has its own semantics of the results [11]. Information uncertainty affects the process and outcomes of information analysis and decision making. There is general agreement that uncertainty affects the decisions we make [10], therefore differences amongst theories are important because they affect the way in which the results should be interpreted and used for decision making.

Although most research dealing with uncertainty visualization has focused on representation, comprehensive guidelines for representing uncertainty do not yet exist. Some recommendations for encoding have been proposed, although little has been done to address the challenge of depicting multiple forms of uncertainty in the same display [10]. For decision making, it is crucial to provide the user with complete information in a suitable form for the best information perception. However cartographic visualizations often fail to do so, because they often presents the data in such form, that it may lead the user to conclusion that the data are actually two-dimensional, with one dimension being the attribute and the second uncertainty. It also points the user to the conclusion that the uncertainty is an attribute of the data, while in fact it is not. Uncertainty is incorporated in each attribute of the data, it is just not commonly displayed. Another issue associated with visualization of the uncertainty is that in many cases it is presented in such a way that it looks like it is symmetrical around some modal value. While this assumption may be true in some cases, it does not generally hold for all the data, especially if the data went through an uncertainty propagation process. After such a calculation, there can be no assumption about the distribution of the uncertainty around modal values, or about existence of some modal value at all. This fact limits the usefulness of some methods of visualization unless they are properly modified for such results.

In the rest of the paper, we would like to show the need of better cartographic approaches in uncertainty visualization.

2 Uncertainty and Decision making

The area of uncertainty is a multidisciplinary topic with very wide span from psychology to mathematics. However, the main research regarding the uncertainty modelling and propagation is done in the field of mathematics [11]. In the same way, the development of methods for decision making with uncertain data is also done in the field of mathematics e.g. [4]. So for working with uncertainty in geosciences the user needs significant knowledge of mathematics and its applications. This is one the reasons why the topics of uncertainty propagation and visualization is rather limited to scientific studies and generally neglected in practical applications.

The uncertainty as understand here is related mainly to inability to specifically proclaim some statement to be exclusively true or false [9]. This can be related to both rational, and ordinal data. In the case of rational data the uncertainty means that the user cannot pinpoint the exact value of the variable, but some values can be specified as being more probable or possible than others, or at least range in which the variable lies can be identified. For ordinal data, the uncertainty means that the user is not able to specify in which category the variable belongs, but again probabilities, possibilities or at least set of possible categories for the data can be specified.

The decision making under uncertainty pose a whole set of problems [2]. But the most important is to present user with three outcomes of the analysis. For user, it is important to know what is best-case, worst-case and most likely result. This poses a problem on its own, because for example method Monte Carlo, which is commonly used in GIS for uncertainty propagation [13], can not guarantee to provide best and worst case results [6], because it is focused on probable outcomes. In general methods based on statistics and probability are often used for uncertainty propagation even though there are many situations when their use is improper or even wrong [6]. In many situations, it is much better to conceptualize the uncertainty by the use of an alternative theory for uncertainty modelling such as interval arithmetic or fuzzy sets [11]. Both these methods can provide estimates of best and worst case results and thus they provide the better framework for modelling and propagation of the uncertainty in many fields including geosciences.

3 Fuzzy surfaces

Fuzzy surface is a surface where each coordinate x, y has associated set of heights, that the surface can have at this location [12]. These possible values of z are represented as a fuzzy numbers $\tilde{Z}$. A fuzzy number is a special case of a fuzzy set, that represents vague, imprecise or ill-known value (Fig. 1) [6]. There are many shapes and form that the fuzzy numbers can, the extensive list of possible definitions is provided in [6]. Three values of the fuzzy number are of interest for decision making, modal (kernel) value and minimal and maximal (support) values.

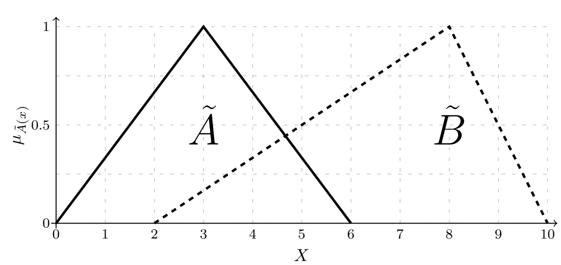

Fig. 1. Symmetrical fuzzy number $\tilde{A}$ and asymmetrical fuzzy number $\tilde{B}$.

Fuzzy numbers can be used in calculations by application of Zadeh's extension principle. This method of uncertainty propagation is called fuzzy arithmetic (sometimes also fuzzy mathematics) [6]. All the operation that can be performed with crisp (classic) numbers can be also done with fuzzy numbers. The calculation with fuzzy numbers produce also fuzzy numbers, so the results naturally contains uncertainty that was in the input values for the calculation. So fuzzy surface can be analysed in the same way as classic surfaces. However so far only relatively few analysis of fuzzy surfaces were actually presented, namely slope [3, 14] and visibility analysis [1].

4 Section visualization of fuzzy surfaces

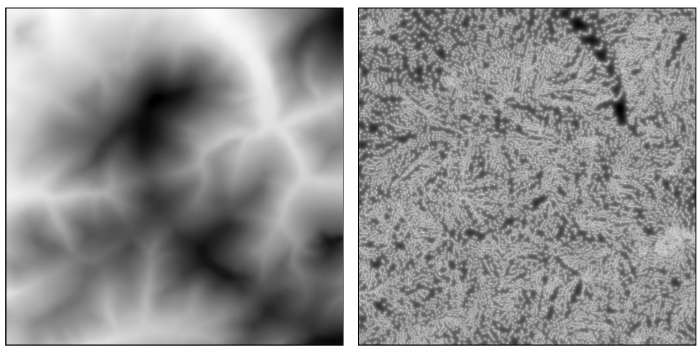

Fig. 2. Modal value of the surface (left), white represents lower values and black higher. Uncertainty of the surface (right), white has lower uncertainty and black is more uncertain.

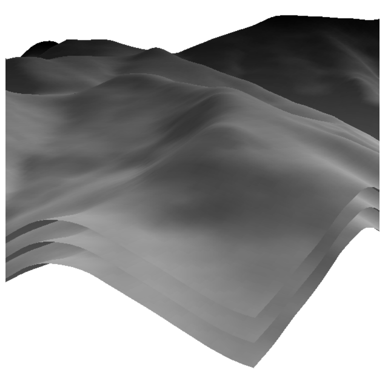

Fig. 3. Visualization of maximal, modal and minimal surface.

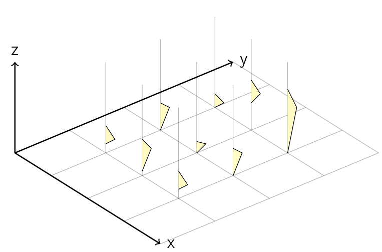

Fig. 4. Visualization of fuzzy numbers from the fuzzy surface.

There are several ways how to visualize fuzzy surface. The classic approach is to visualize modal value of the surface and the uncertainty as two images (Fig. 2). The problem is that it is rather hard to relate these two information together because they are shown as separated images.

Besides that fuzzy profile was shown in [9], however such visualization shows only portion of the whole surface. In [12] visualizations using 3 surfaces (Fig. 3) and direct visualization of fuzzy numbers (Fig. 4) is shown. Neither is good for the user, the former is impossible to interpret and the latter is only interpretable for very small surfaces. If the surface is larger in its spatial extent it is almost impossible to read anything useful from it.

5 Proposed approach

Possible way to visualize fuzzy surface, or result of fuzzy arithmetic operation on the surface is in use of the HSL color model [5]. Suppose that we have set of fuzzy numbers $\tilde{Z}_1, \dots, \tilde{Z}_n$ that form the fuzzy surface. For each fuzzy number $\tilde{Z}_i$ there are values of $\tilde{Z}_i^-$ representing minimum, $\tilde{Z}_i^m$ representing modal value and $\tilde{Z}_i^+$ for maximum. The range of values $\min(\tilde{Z}_1^m, \dots, \tilde{Z}_n^m)$, $\max(\tilde{Z}_1^m, \dots, \tilde{Z}_n^m)$ can be used to map the value of H to each cell of this fuzzy surface in the same way as classic colour scales are mapped for visualization. For each fuzzy number the values of $nu_i = \tilde{Z}_i^m - \tilde{Z}_i^-$ and $pu_i = \tilde{Z}_i^+ - \tilde{Z}_i^m$ can be calculated. These denotes negative uncertainty (nu) of the fuzzy number and positive uncertainty (pu) and the show magnitude of the difference between extreme values and the modal value.

The values of L in the color model denotes of the colour is whiter or darker, some midpoint (usually value 0.5) denotes neutral colours. We can use value of nu_i that is substracted from the midpoint to deviate the colour towards white to show magnitude of uncertainty towards lower values. In the same way, the values of pu_i added to the midpoint sway the colour towards black to show the magnitude towards black (Figs. 5). Values of parameters S of the model is fixed. The approach can be seen as an extension of the approach proposed by [7].

6 Conclusion

Figure 7 shows the visualization of the fuzzy surface. The hue allows user to estimate modal value of the fuzzy surface, and whiteness and darkness provide him with estimation of minimal and maximal value. Verification of the proposed methods suitability are broad from the survey of respondents' answers to specific questions, to the use of most advanced technologies in the field of cognitive cartography. Information perception of fuzzy surfaces visualizations made on the basis of internal graphic variables HSL model will be evaluated by the eye-tracking system in next step. The general perception of geovisualization is also significantly affected by a particular user, the knowledge, skills and other characteristics, but the main aim of the researcher is to provide as accurate information as possible.

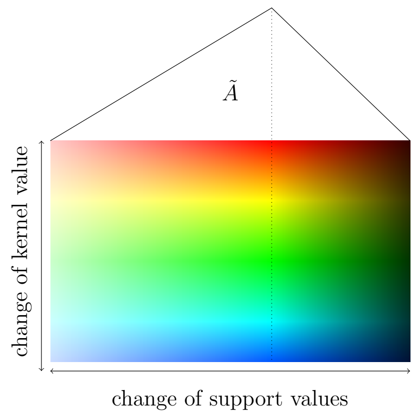

Fig. 5. Simple visualization of proposed approach for visualization of fuzzy surface.

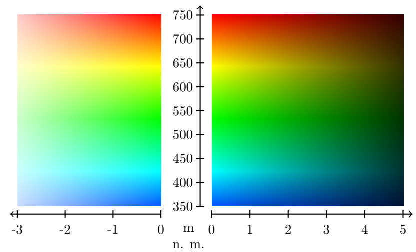

Fig. 6. Visualization of possible legend for the visualization of the fuzzy surface.

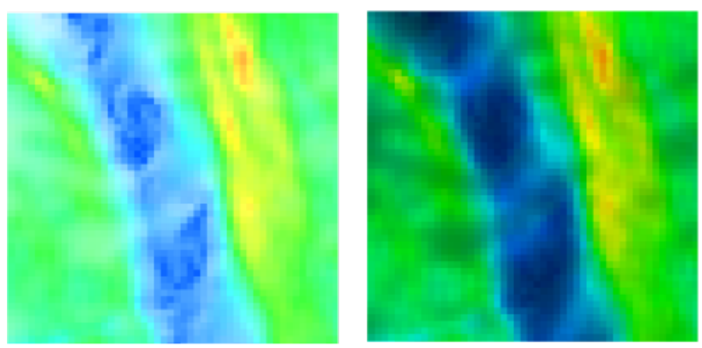

Fig. 7. Visualization of fuzzy surface using legend from Fig. 6.

The proposed approach significantly improves the readability of the information about fuzzy surface.

Acknowledgement

The authors gratefully acknowledge the support by the Operational Program Education for Competitiveness - European Social Fund (project CZ.1.07/2.3.00/20.0170 of the Ministry of Education, Youth and Sports of the Czech Republic) and project IGA_PrF_2014007 of the Palacký University, Olomouc.

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